

Computing Reverse Interval Power Functions

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Scan 2012

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- Difficulties with inversion of interval power functions
 - Overvalue
 - Wrong

$$x, y, z \in \mathbb{R}, x > 0, y \neq 0$$

$$x^y = z$$

$$\Rightarrow x = z^{1/y}$$

Problem

$$y = [-0.5, 0.4], z = [0.25, 0.25]$$

$$z^{1/y} = z^{]-\infty, +\infty[} = [0, +\infty[$$

$$\neq]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\underline{y}}, +\infty[=]0, 0.03125] \cup [16, +\infty[$$

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Definition

For a (partial) binary arithmetic operation \circ there are two *binary reverse operations* on intervals,

$$\circ_1^- : \overline{\mathbb{R}} \times \overline{\mathbb{R}} \rightarrow \wp(\mathbb{R})$$

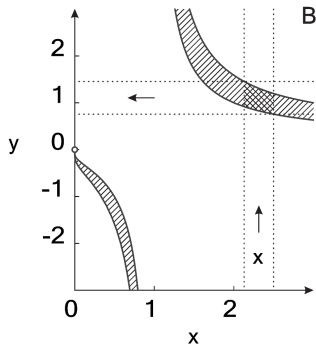
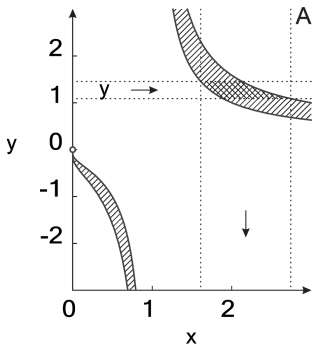
$$\circ_2^- : \overline{\mathbb{R}} \times \overline{\mathbb{R}} \rightarrow \wp(\mathbb{R})$$

defined by

$$\circ_1^-(\mathbf{y}, \mathbf{z}) = \{x \in \mathbb{R} \mid \text{there exists } y \in \mathbf{y} \text{ with } x \circ y \in \mathbf{z}\}$$

$$\circ_2^-(\mathbf{x}, \mathbf{z}) = \{y \in \mathbb{R} \mid \text{there exists } x \in \mathbf{x} \text{ with } x \circ y \in \mathbf{z}\}$$

with $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \overline{\mathbb{R}}$



$$\circ_1^-(\mathbf{y}, \mathbf{z}) = \{x \in \mathbb{R} \mid \text{there exists } y \in \mathbf{y} \text{ with } x \circ y \in \mathbf{z}\}$$

$$\circ_2^-(\mathbf{x}, \mathbf{z}) = \{y \in \mathbb{R} \mid \text{there exists } x \in \mathbf{x} \text{ with } x \circ y \in \mathbf{z}\}$$

Definition

For a (partial) binary arithmetic operation \circ there are two *ternary reverse operations* on intervals,

$$\bullet_1^- : \overline{\mathbb{R}} \times \overline{\mathbb{R}} \times \overline{\mathbb{R}} \rightarrow \overline{\mathbb{R}}$$

$$\bullet_2^- : \overline{\mathbb{R}} \times \overline{\mathbb{R}} \times \overline{\mathbb{R}} \rightarrow \overline{\mathbb{R}}$$

defined by

$$\bullet_1^-(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \text{hull}(\{x \in \mathbf{x} \mid \text{there exists } y \in \mathbf{y} \text{ with } x \circ y \in \mathbf{z}\})$$

$$\bullet_2^-(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \text{hull}(\{y \in \mathbf{y} \mid \text{there exists } x \in \mathbf{x} \text{ with } x \circ y \in \mathbf{z}\})$$

with $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \overline{\mathbb{R}}$

$$\bullet \bar{o}_1(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathit{hull}(\mathbf{x} \cap o_1^-(\mathbf{y}, \mathbf{z}))$$

$$\bullet \bar{o}_2(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathit{hull}(\mathbf{y} \cap o_2^-(\mathbf{x}, \mathbf{z}))$$

⇒ It is sufficient to present reverse operations o_1^- and o_2^- .

$$\bullet \bar{o}_1(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathit{hull}(\mathbf{x} \cap o_1^-(\mathbf{y}, \mathbf{z}))$$

$$\bullet \bar{o}_2(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathit{hull}(\mathbf{y} \cap o_2^-(\mathbf{x}, \mathbf{z}))$$

⇒ It is sufficient to present reverse operations o_1^- and o_2^- .

Name	Domain	Computation
pow1	$\mathbb{R}^+ \times \mathbb{R}$	$(x, y) \mapsto \exp(y \cdot \log x)$
pow2	$\mathbb{R}^+ \times \mathbb{R}$	$(x, y) \mapsto \exp(y \cdot \log x)$
	$\{0\} \times \mathbb{R}^+$	$(x, y) \mapsto 0$
	$\mathbb{R}^- \times \mathbb{Z}$	$(x, y) \mapsto \begin{cases} \exp(y \cdot \log x) & \text{if } y \text{ even} \\ -\exp(y \cdot \log x) & \text{if } y \text{ odd} \end{cases}$
pow3	$\mathbb{R}^+ \times \mathbb{R}$	$(x, y) \mapsto \exp(y \cdot \log x)$
	$\{0\} \times \mathbb{R}^+$	$(x, y) \mapsto 0$
	$\mathbb{R}^- \times D$	$(x, \frac{m}{n}) \mapsto \begin{cases} \exp(\frac{m}{n} \cdot \log x) & \text{if } m \text{ even} \\ & \text{and } n \text{ odd} \\ -\exp(\frac{m}{n} \cdot \log x) & \text{if } m \text{ odd} \\ & \text{and } n \text{ odd} \end{cases}$
		with $D = \{\frac{m}{n} \mid m, n \in \mathbb{Z}, n \text{ odd}\}$

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$$\text{pow1} \quad \mathbb{R}^+ \times \mathbb{R} \quad (x, y) \mapsto \exp(y \cdot \log x)$$

- Interval overlapping \oplus
- $z \in [0, 1]$

\Rightarrow Eight groups of images on real intervals z

$$\text{pow1} \quad \mathbb{R}^+ \times \mathbb{R} \quad (x, y) \mapsto \exp(y \cdot \log x)$$

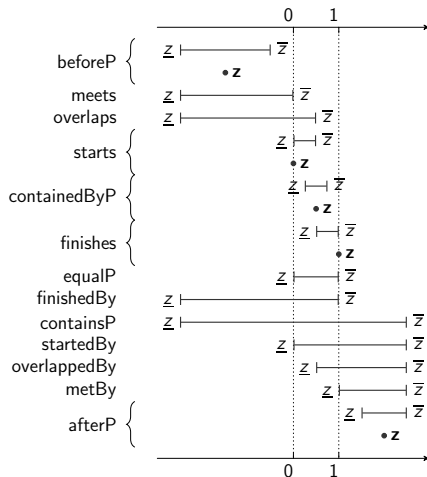
- Interval overlapping \oplus
- $\mathbf{z} \oplus [0, 1]$

\Rightarrow Eight groups of images on real intervals \mathbf{z}

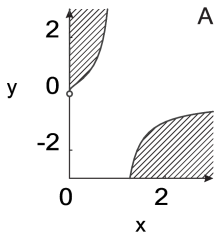
Interval overlapping

Sketches for the 13 states

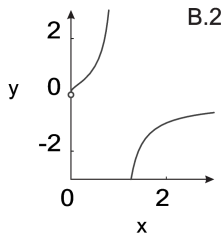
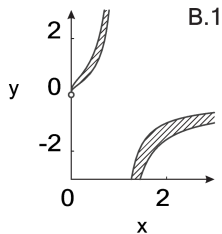
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Overlapping states $z \in [0, 1]$ with non-empty, bound $z = [z, \bar{z}]$.



- $z \in [0, 1]$
 - overlaps
 - starts



■ $z \in [0, 1]$

■ containedByP

$\text{pow1}_1^-(\mathbf{y}, \mathbf{z})$:

$\mathbf{z} \in [0, 1]$	$\mathbf{y} \in [0, 0]$					
	beforeP	equalP	finishedBy	containsP	startedBy	afterP
overlaps/starts	$[\bar{z}^{1/\bar{y}}, +\infty[$	\emptyset	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$	$]0, \bar{z}^{1/\bar{y}}]$
containedByP	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	\emptyset	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$
finishes	$]1, \bar{z}^{1/\bar{y}}]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$[\bar{z}^{1/\bar{y}}, 1]$
equalP/finishedBy	$]1, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, 1]$
containsP/startedBy	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$
overlappedBy	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$
metBy	$[\bar{z}^{1/\bar{y}}, 1]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]1, \bar{z}^{1/\bar{y}}]$
afterP	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	\emptyset	$]0, \bar{z}^{1/\bar{y}}]$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$[\bar{z}^{1/\bar{y}}, +\infty[$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$

Note: For $\bar{z} \leq 0$ the result is \emptyset . Results for unbounded intervals can easily be obtained via limit values.

- $\text{pow1}_2^-(\mathbf{x}, \mathbf{z})$ is defined in a similar manner
- Ternary function uses hull

$\text{pow1}_1^-(\mathbf{y}, \mathbf{z})$:

$\mathbf{z} \in [0, 1]$	$\mathbf{y} \in [0, 0]$					
	beforeP	equalP	finishedBy	containsP	startedBy	afterP
overlaps/starts	$[\bar{z}^{1/\bar{y}}, +\infty[$	\emptyset	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$	$]0, \bar{z}^{1/\bar{y}}]$
containedByP	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	\emptyset	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$
finishes	$]1, \bar{z}^{1/\bar{y}}]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$[\bar{z}^{1/\bar{y}}, 1]$
equalP/finishedBy	$]1, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, 1]$
containsP/startedBy	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$
overlappedBy	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$
metBy	$[\bar{z}^{1/\bar{y}}, 1]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]1, \bar{z}^{1/\bar{y}}]$
afterP	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	\emptyset	$]0, \bar{z}^{1/\bar{y}}]$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$[\bar{z}^{1/\bar{y}}, +\infty[$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$

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$\mathbf{z} \in [0, 1]$	$\mathbf{y} \in [0, 0]$					
	beforeP	equalP	finishedBy	containsP	startedBy	afterP
overlaps/starts	$[\bar{z}^{1/\bar{y}}, +\infty[$	\emptyset	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$	$]0, \bar{z}^{1/\bar{y}}]$
containedByP	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	\emptyset	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$
finishes	$]1, \bar{z}^{1/\bar{y}}]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$[\bar{z}^{1/\bar{y}}, 1]$
equalP/finishedBy	$]1, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, 1]$
containsP/startedBy	$[\bar{z}^{1/\bar{y}}, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, \bar{z}^{1/\bar{y}}]$
overlappedBy	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$
metBy	$[\bar{z}^{1/\bar{y}}, 1]$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]0, +\infty[$	$]1, \bar{z}^{1/\bar{y}}]$
afterP	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$	\emptyset	$]0, \bar{z}^{1/\bar{y}}]$	$]0, \bar{z}^{1/\bar{y}}] \cup [\bar{z}^{1/\bar{y}}, +\infty[$	$[\bar{z}^{1/\bar{y}}, +\infty[$	$[\bar{z}^{1/\bar{y}}, \bar{z}^{1/\bar{y}}]$

Note: For $\bar{z} \leq 0$ the result is \emptyset . Results for unbounded intervals can easily be obtained via limit values.

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- Ternary function uses hull

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$$\begin{array}{l}
 \text{pow2} \\
 \mathbb{R}^+ \times \mathbb{R} \quad (x, y) \mapsto \exp(y \cdot \log x) \\
 \{0\} \times \mathbb{R}^+ \quad (x, y) \mapsto 0 \\
 \mathbb{R}^- \times \mathbb{Z} \quad (x, y) \mapsto \begin{cases} \exp(y \cdot \log |x|) & \text{if } y \text{ even} \\ -\exp(y \cdot \log |x|) & \text{if } y \text{ odd} \end{cases}
 \end{array}$$

■ Special interval overlapping $\circ\circ$

- $\circ\circ : ([\underline{a}, \bar{a}], [\underline{b}, \bar{b}]) \mapsto ([\underline{a}, \bar{a}] \circ [\underline{b}, m], [\underline{a}, \bar{a}] \circ [m, \bar{b}])$
- 26 cases

■ $z \circ\circ[-1, 1]$

\Rightarrow Look-up-table

$$\text{pow2} \quad \begin{array}{ll} \mathbb{R}^+ \times \mathbb{R} & (x, y) \mapsto \exp(y \cdot \log x) \\ \{0\} \times \mathbb{R}^+ & (x, y) \mapsto 0 \\ \mathbb{R}^- \times \mathbb{Z} & (x, y) \mapsto \begin{cases} \exp(y \cdot \log |x|) & \text{if } y \text{ even} \\ -\exp(y \cdot \log |x|) & \text{if } y \text{ odd} \end{cases} \end{array}$$

- Special interval overlapping $\circ\circ$

- $\circ\circ : ([\underline{a}, \bar{a}], [\underline{b}, \bar{b}]) \mapsto ([\underline{a}, \bar{a}] \circ [\underline{b}, m], [\underline{a}, \bar{a}] \circ [m, \bar{b}])$

- 26 cases

- $\mathbf{z} \circ\circ[-1, 1]$

\Rightarrow Look-up-table

Special interval overlapping

Definition of the 26 states

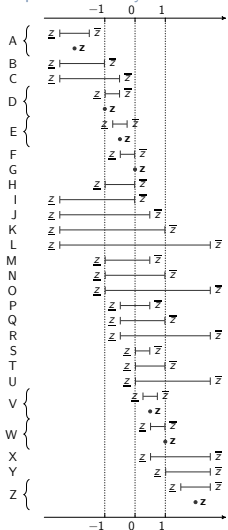
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$[a, \bar{a}] \circlearrowleft [b, \bar{b}]$	Condition	
	$[a, \bar{a}] \circlearrowleft [b, m]$	$[a, \bar{a}] \circlearrowleft [m, b]$
A	beforeP	beforeP
B	meets	beforeP
C	overlaps	beforeP
D	starts	beforeP
E	containedByP	beforeP
F	finishes	meets
	⋮	
V	afterP	containedByP
W	afterP	finishes
X	afterP	overlappedBy
Y	afterP	metBy
Z	afterP	afterP

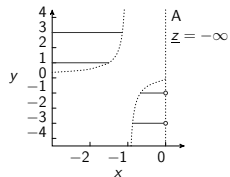
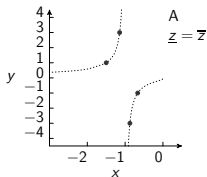
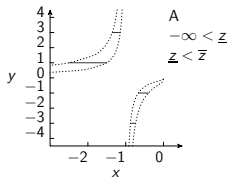
Special interval overlapping

Sketches for the 26 states

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Overlapping states $z \in [-1, 1]$ with non-empty, bound $z = [z, \bar{z}]$.



Require: x, y, z

Ensure: $\mathbf{h} = \text{pow2}_1^-(x, y, z) = \text{hull}(x \cap \text{pow2}_1^-(y, z))$

1: $\mathbf{h}^+ \leftarrow \text{pow1}_1^-(x, y, z)$

2: $\mathbf{x}^- \leftarrow x \cap] - \infty, 0]$

3: $\mathbf{h}^- \leftarrow \text{hull}(\mathbf{x}^- \cap \text{pow2}_1^-(y, z))$ with Look-up-table

4: $\mathbf{h} \leftarrow \text{hull}(\mathbf{h}^- \cup \mathbf{h}^+)$

■ pow2_2^- is defined in a similar manner

Require: x, y, z

Ensure: $h = \text{pow2}_1^-(x, y, z) = \text{hull}(x \cap \text{pow2}_1^-(y, z))$

1: $h^+ \leftarrow \text{pow1}_1^-(x, y, z)$

2: $x^- \leftarrow x \cap] - \infty, 0]$

3: $h^- \leftarrow \text{hull}(x^- \cap \text{pow2}_1^-(y, z))$ with Look-up-table

4: $h \leftarrow \text{hull}(h^- \cup h^+)$

■ pow2_2^- is defined in a similar manner

- Some difficult cases for pow_1^-
 - $z \in [-1, 1]$
 - A, E
 - V, Z
- Union of infinitely many disjoint intervals

$$\begin{aligned}
 \text{pow}_1^-([1, +\infty], [2, 3]) &= \text{pow}_1^-([1, +\infty], [2, 3]) \cup \bigcup_{\substack{n \in [1, +\infty] \\ n \text{ even}}} [-3^{1/n}, -2^{1/n}] \\
 &\subseteq \text{pow}_1^-([1, +\infty], [2, 3]) \cup \\
 &\quad \underbrace{[-1.74, -1.41] \cup [-1.32, -1.18] \cup \dots}_{\text{disjoint}}
 \end{aligned}$$

Problem

$$\text{pow}_1^-(x, y, z) = \text{hull}(x \cap \text{pow}_1^-(y, z))$$

- Some difficult cases for $\text{pow}2_1^-$
 - $z \in]-1, 1[$
 - A, E
 - V, Z
- Union of infinitely many disjoint intervals

$$\begin{aligned}
 \text{pow}2_1^-([1, +\infty], [2, 3]) &= \text{pow}1_1^-([1, +\infty], [2, 3]) \cup \bigcup_{\substack{n \in [1, +\infty] \\ n \text{ even}}} [-3^{1/n}, -2^{1/n}] \\
 &\subseteq \text{pow}1_1^-([1, +\infty], [2, 3]) \cup \\
 &\quad \underbrace{[-1.74, -1.41] \cup [-1.32, -1.18] \cup \dots}_{\text{disjoint}}
 \end{aligned}$$

Problem

$$\text{pow}2_1^-(x, y, z) = \text{hull}(x \cap \text{pow}2_1^-(y, z))$$

Require: $x = [\underline{x}, \bar{x}]$ with $\bar{x} \leq 0$, $y = [\underline{y}, \bar{y}]$ with $\underline{y} > 0$, $z = [\underline{z}, \bar{z}]$ with $\underline{z} > 1$

Ensure: $[\underline{r}, \bar{r}] = \text{pow2}_1^-(x, y, z) = x \cap \bigcup_{\substack{n \in \mathbb{Y} \\ n \text{ even}}} [-|\bar{z}|^{1/n}, -|\underline{z}|^{1/n}]$

1: $[\underline{h}, \bar{h}] \leftarrow [-|\bar{z}|^{1/lee}, -|\underline{z}|^{1/gee}]$ {enclosure of union of intervals}

2: $[\underline{r}, \bar{r}] \leftarrow [\underline{x}, \bar{x}] \cap [\underline{h}, \bar{h}]$ {enclosure of result}

3: **if** $\underline{h} < \underline{x} < \bar{h}$ **then**

4: {optimize left boundary of result}

5: $a \leftarrow -\log_{|\underline{x}|} |\underline{z}|$

6: $b \leftarrow \min\{y \in \mathbb{Z} \mid y \text{ even and } y \geq a\}$

7: $c \leftarrow -|\bar{z}|^{1/b}$

8: $\underline{r} \leftarrow \max\{\underline{r}, c\}$

9: **end if**

10: **if** $\underline{h} < \bar{x} < \bar{h}$ **then**

11: {optimize right boundary of result}

12: $a \leftarrow -\log_{|\bar{x}|} |\bar{z}|$

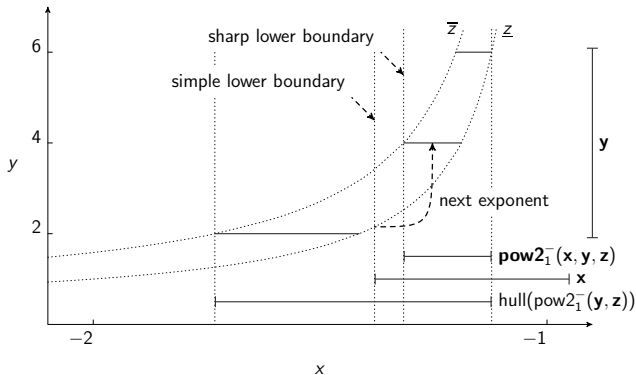
13: $b \leftarrow \max\{y \in \mathbb{Z} \mid y \text{ even and } y \leq a\}$

14: $c \leftarrow -|\underline{z}|^{1/b}$

15: $\bar{r} \leftarrow \min\{\bar{r}, c\}$

16: **end if**

17: {if $\underline{r} > \bar{r}$, the result is the empty set}



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pow3

$$\begin{aligned} \mathbb{R}^+ \times \mathbb{R} & \quad (x, y) \mapsto \exp(y \cdot \log x) \\ \{0\} \times \mathbb{R}^+ & \quad (x, y) \mapsto 0 \\ \mathbb{R}^- \times D & \quad (x, \frac{m}{n}) \mapsto \begin{cases} |x|^{m/n} & \text{if } m \text{ even, } n \text{ odd} \\ -|x|^{m/n} & \text{if } m \text{ odd, } n \text{ odd} \end{cases} \end{aligned}$$

with $D = \{\frac{m}{n} \mid m, n \in \mathbb{Z}, n \text{ odd}\}$

■ Put together using reverse pow1

pow3

$$\begin{aligned} \mathbb{R}^+ \times \mathbb{R} & (x, y) \mapsto \exp(y \cdot \log x) \\ \{0\} \times \mathbb{R}^+ & (x, y) \mapsto 0 \\ \mathbb{R}^- \times D & (x, \frac{m}{n}) \mapsto \begin{cases} |x|^{m/n} & \text{if } m \text{ even, } n \text{ odd} \\ -|x|^{m/n} & \text{if } m \text{ odd, } n \text{ odd} \end{cases} \end{aligned}$$

with $D = \{\frac{m}{n} \mid m, n \in \mathbb{Z}, n \text{ odd}\}$

- Put together using reverse pow1

$$\begin{aligned}
 \text{pow3}_1^-(\mathbf{y}, \mathbf{z}) &= \text{pow1}_1^-(\mathbf{y}, \mathbf{z}) \\
 &\cup \begin{cases} \{0\} & \text{if } \mathbf{y} \cap \mathbb{R}^+ \neq \emptyset \text{ and } 0 \in \mathbf{z} \\ \emptyset & \text{otherwise} \end{cases} \\
 &\cup -\text{pow1}_1^-(\mathbf{y}_{\text{even}}, \mathbf{z}) \\
 &\cup -\text{pow1}_1^-(\mathbf{y}_{\text{odd}}, -\mathbf{z}),
 \end{aligned}$$

where

$$\begin{aligned}
 \mathbb{Q}_{\text{even}} &= \{r \in \mathbb{Q} \mid r = \frac{m}{n} \text{ with } m \text{ even and } n \text{ odd}\}, \\
 \mathbb{Q}_{\text{odd}} &= \{r \in \mathbb{Q} \mid r = \frac{m}{n} \text{ with } m \text{ odd and } n \text{ odd}\}, \\
 \mathbf{y}_{\text{even}} &= \text{hull}(\mathbf{y} \cap \mathbb{Q}_{\text{even}}) \\
 \mathbf{y}_{\text{odd}} &= \text{hull}(\mathbf{y} \cap \mathbb{Q}_{\text{odd}})
 \end{aligned}$$

$$\text{pow3}_2^-(\mathbf{x}, \mathbf{z}) = \text{pow1}_2^-(\mathbf{x}, \mathbf{z})$$

$$\cup \begin{cases} \mathbb{R}^+ & \text{if } 0 \in \mathbf{x} \text{ and } 0 \in \mathbf{z} \\ \emptyset & \text{otherwise} \end{cases}$$

$$\cup (\text{pow1}_2^-(-\mathbf{x}, \mathbf{z}) \cap \mathbb{Q}_{\text{even}})$$

$$\cup (\text{pow1}_2^-(-\mathbf{x}, -\mathbf{z}) \cap \mathbb{Q}_{\text{odd}}),$$

where

$$\mathbb{Q}_{\text{even}} = \left\{ r \in \mathbb{Q} \mid r = \frac{m}{n} \text{ with } m \text{ even and } n \text{ odd} \right\},$$

$$\mathbb{Q}_{\text{odd}} = \left\{ r \in \mathbb{Q} \mid r = \frac{m}{n} \text{ with } m \text{ odd and } n \text{ odd} \right\},$$

- 1 Introduction
- 2 Reverse pow1
- 3 Reverse pow2
- 4 Reverse pow3
- 5 Summary**

- Reverse interval power functions
- Definition
- Specification
- Implementation

Questions ?

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